

## CLAIMS

What is claimed is:

1. A method for solving a Model Predictive Control problem including the steps of:

a) forming a Large Sparse Matrix Equation, LSME, based upon the Model Predictive Control problem;

b) repeatedly, at least once per controller update, sampling commands and feedback sensors to pose a determination of actuator commands in terms of a solution of a quadratic programming problem based upon model predictive control;

c) forming a square root of  $H$ ,  $H_r$ , where  $H$  is a submatrix of the LSME that varies only once per controller update and  $H_r$  is a block triangular matrix in which each block along a diagonal is triangular;

d) forming a square root of a Large Sparse Matrix of the LSME,  $LSM_{root}$ , using  $H_r$  in each of a plurality of iterations of a quadratic programming solver within each controller update, and  $LSM_{root}$  is block triangular matrix in which each block along the diagonal is triangular; and

e) completing the solution of the LSME based upon  $LSM_{root}$  in each iteration.

2. The method of claim 1 wherein  $H_r$  is calculated in a first of the plurality of iterations and is not recalculated for subsequent ones of the plurality of iterations.

3. The method of claim 2 wherein in said step c),  $H_r$  is formed without first forming  $H$ .

4. The method of claim 3 wherein said step c) is performed with a sequence of QR factorizations.

5. The method of claim 3 where  $H = H_r' * S * H_r$  and  $H_r = L W^T$  where

$$L = \begin{bmatrix} F_0 & & & & & & & & & & \\ -M_1 B_0 & I & & & & & & & & & \\ 0 & I & -M_1 & & & & & & & & \\ & 0 & G_1 & F_1 & & & & & & & \\ & & -M_2 A_1 & -M_2 B_1 & I & & & & & & \\ & & & 0 & I & -M_2 & & & & & \\ & & & & \ddots & & & & & & \\ & & & & & 0 & G_{N-1} & F_{N-1} & & & \\ & & & & & & -M_N A_{N-1} & -M_N B_{N-1} & I & & \\ & & & & & & & 0 & I & -M_N & \end{bmatrix}$$

$$S = \begin{bmatrix} I & & & & & & & & & & \\ & -I & & & & & & & & & \\ & & I & & & & & & & & \\ & & & I & & & & & & & \\ & & & & -I & & & & & & \\ & & & & & I & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & I & & & \\ & & & & & & & & -I & & \\ & & & & & & & & & I & \end{bmatrix}$$

$$W = \begin{bmatrix} I & & & & & & & & & & \\ & M_1 & & & & & & & & & \\ & & I & & & & & & & & \\ & & & I & & & & & & & \\ & & & & M_2 & & & & & & \\ & & & & & I & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & I & & & \\ & & & & & & & & M_N & & \\ & & & & & & & & & I & \end{bmatrix}$$

6. The method of claim 5 where the submatrices F, G, and M are produced by the recursive sequence of QR factorizations:

$$\begin{bmatrix} Q^{1/2} C_n & Q^{1/2} D_n \\ M_n A_{n-1} & M_n B_{n-1} \\ 0 & R^{1/2} \end{bmatrix} = Ortho * \begin{bmatrix} 0 & 0 \\ M_{n-1} & 0 \\ -G_{n-1} & F_{n-1} \end{bmatrix}$$

for various future controller update time points in the prediction horizon.

7. The method of claim 1 wherein the LSMroot is formed in said step d) without first forming the LSM.
8. The method of claim 7 wherein said step d) is performed by solving a normal or generalized QR factorization.
9. The method of claim 7 wherein said step d) is performed by solving a generalized QR factorization customized for an interior point method.
10. The method of Claim 9 where the generalized QR factorization is

$$\begin{bmatrix} 0 \\ P \end{bmatrix} = U \cdot Pin, \text{ where } Pin = \begin{bmatrix} T^{-T} J \\ L \end{bmatrix}, U^T \begin{bmatrix} I \\ S \end{bmatrix} U = \begin{bmatrix} I \\ S \end{bmatrix}$$

where Pin and S are inputs and triangular P and, possibly, U are outputs, and where

$$LSM_{root} \equiv \begin{bmatrix} T & -T^{-T} J \\ 0 & P \end{bmatrix} \cdot \begin{bmatrix} I \\ W^{-T} \end{bmatrix} \cdot \text{and } Sr=I.$$

11. The method of claim 7 wherein said step d) is performed by solving a generalized QR factorization customized for an active set method.
12. The method of Claim 11 where the generalized QR factorization is

$$S_R^{1/2} P = S^{1/2} U \cdot Pin, \text{ where } Pin = L^{-T} J^T, U^T S \cdot U = S$$

where Pin and S are inputs and P and Sr, and possibly, U are outputs and where

$$LSM_{root} = \begin{bmatrix} P & 0 \\ SL^{-T}J^T & L \end{bmatrix} \begin{bmatrix} I \\ W^{-T} \end{bmatrix}$$

13. The method of Claim 12 where the generalized QR factorization is accomplished via a standard Householder transform based QR algorithm that is modified by replacing the sequence of Householder transforms

$$U_i = \left( I - 2 \frac{v \cdot v^T}{v^T \cdot v} \right), \text{ which are normally used with a corresponding sequence}$$

of generalized transformations  $U_i = \left( I - 2 \frac{v \cdot v^T S}{v^T S \cdot v} \right)$ . U is unneeded but, if desired, is the cumulative product of the  $U_i$ .

14. The method of claim 13 where the result of generalized QR factorization, which is a triangular matrix consisting of rows containing either purely real numbers or purely imaginary numbers, is further factored into the product of a diagonal matrix,  $S_R^{1/2}$ , which has either 1 or the square root of  $-1$  on the diagonal, and a purely real triangular matrix, and wherein the square of this diagonal matrix,  $S_r$ , will then have either 1 or  $-1$  on the diagonal, and  $S_r$  and  $P$  are purely real numbers.

15. The method of claim 11 further including the step of factoring the LSM to use only real numbers as:

$$LSM = LSM_{root}^T \begin{bmatrix} -S_R & 0 \\ 0 & S \end{bmatrix} LSM_{root}$$

where S and SR are constant diagonal matrices with only 1 or -1 entries along the diagonal, and where LSMroot is a block triangular matrix where each diagonal submatrix is also triangular.

16. The method of claim 1 wherein the LSME has the form:

$$\begin{bmatrix} -T^T T & J \\ J^T & H \end{bmatrix} \begin{bmatrix} m \\ z \end{bmatrix} = \begin{bmatrix} K \\ f \end{bmatrix}$$

where z is a vector containing states, controls and state equation adjoint variables for each time point, grouped by time; m includes adjoint variables for inequality constraints, grouped by time; f and K are vectors; H and J are banded matrices; and T is a diagonal matrix or zero, depending on the quadratic program algorithm selected.

17. A method for controlling a system including the steps of:
- a) Receiving a plurality of sensor signals indicating current conditions of the system;
  - b) Receiving a plurality of commands;
  - c) Determining a desired dynamic response of the system based upon the commands;
  - d) in each of a plurality of time steps, formulating a problem of achieving a desired dynamic response for a window spanning multiple time steps as a solution to a quadratic program problem using methods of model predictive control and the sensor signals;
  - e) solving the quadratic programming problem in each time step using an iterative algorithm in which a Large Sparse Matrix Equation, LSME, is formed based upon the model predictive control problem and a quadratic programming algorithm;
  - f) in a first iteration of the iterative algorithm, forming a square root of  $H$ ,  $H_r$ ;
  - g) in iterations including the first iteration and iterations subsequent to the first iteration of the iterative algorithm, forming a square root of a Large Sparse Matrix of the LSME,  $LSM_{root}$ , using  $H_r$  in a quadratic programming solver; and
  - h) in iterations including the first iteration and iterations subsequent to the first iteration of the iterative algorithm, completing the solution of the LSME based upon  $LSM_{root}$ .

18. A model predictive control system for controlling a plant comprising:

- a plurality of sensors indicating a current state of the system;
- a desired trajectory generator for creating a desired dynamic response based upon commands;
- starting with a current state of the system, a quadratic programming module formulating a problem of achieving the desired dynamic response for a window spanning multiple time steps as a solution to a quadratic program problem using methods of model predictive control;
- a quadratic programming solver solving the quadratic programming problem in each time step using an iterative algorithm in which a Large Sparse Matrix Equation, LSME, is formed based upon the model predictive control problem and a quadratic programming algorithm, the solver forming a square root of  $H$ ,  $H_r$ , in a first iteration of the iterative algorithm, the solver forming a square root of a Large Sparse Matrix of the LSME,  $LSM_{root}$ , using  $H_r$  in a quadratic programming solver in iterations including the first iteration and a plurality of iterations subsequent to the first iteration of the iterative algorithm, the solver completing the solution of the LSME based upon  $LSM_{root}$  in iterations including the first iteration and the plurality of iterations subsequent to the first iteration of the iterative algorithm.

19. The system of claim 18 wherein the solver forms  $H_r$  in the first iteration and does not recalculate  $H_r$  in the plurality of iterations subsequent to the first iteration of the iterative algorithm.

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23. The system of claim 18 wherein the solver forms LSMroot without first forming the LSM.

24. The system of claim 23 wherein the solver forms the LSMroot by solving a normal or generalized QR factorization.

25. The system of claim 23 wherein the solver forms the LSMroot by solving a generalized QR factorization customized for an interior point method.

26. The system of claim 25 where the generalized QR factorization is

$$\begin{bmatrix} 0 \\ P \end{bmatrix} = U \cdot Pin, \text{ where } Pin = \begin{bmatrix} T^{-T} J \\ L \end{bmatrix}, U^T \begin{bmatrix} I \\ S \end{bmatrix} U = \begin{bmatrix} I \\ S \end{bmatrix}$$

where Pin and S are inputs and triangular P and, possibly, U are outputs, and where

$$LSM_{root} \equiv \begin{bmatrix} T & -T^{-T} J \\ 0 & P \end{bmatrix} \cdot \begin{bmatrix} I \\ W^{-T} \end{bmatrix} \cdot \text{and } Sr=I.$$

27. The system of claim 23 wherein the solver forms LSMroot by solving a generalized QR factorization customized for an active set method.

28. The system of Claim 27 where the generalized QR factorization is

$$S_R^{1/2} P = S^{1/2} U \cdot Pin, \text{ where } Pin = L^{-T} J^T, U^T S \cdot U = S$$

where Pin and S are inputs and P and Sr, and possibly, U are outputs and where

$$LSM_{root} = \begin{bmatrix} P & 0 \\ SL^{-T}J^T & L \end{bmatrix} \begin{bmatrix} I \\ W^{-T} \end{bmatrix}$$

29. The system of Claim 28 where the generalized QR factorization is accomplished via a standard Householder transform based QR algorithm that is modified by replacing the sequence of Householder transforms

$$U_i = \left( I - 2 \frac{v \cdot v^T}{v^T \cdot v} \right), \text{ which are normally used with a corresponding sequence}$$

$$\text{of generalized transformations } U_i = \left( I - 2 \frac{v \cdot v^T S}{v^T S \cdot v} \right). \text{ U is unneeded but, if}$$

desired, is the cumulative product of the  $U_i$ .

30. The system of claim 29 where the result of generalized QR factorization, which is a triangular matrix consisting of rows containing either purely real numbers or purely imaginary numbers, is further factored into the product of a diagonal matrix,  $S_R^{1/2}$ , which has either 1 or the square root of  $-1$  on the diagonal, and a purely real triangular matrix and wherein a square of this diagonal matrix,  $S_r$ , will then have either 1 or  $-1$  on the diagonal, and  $S_r$  and  $P$  are purely real numbers.

31. The system of claim 27 wherein the solver factors the LSM to use only real numbers as:

$$LSM = LSM_{root}^T \begin{bmatrix} -S_R & 0 \\ 0 & S \end{bmatrix} LSM_{root}$$

where S and SR are constant diagonal matrices with only 1 or -1 entries along the diagonal, and where LSMroot is a block triangular matrix where each diagonal submatrix is also triangular.

32. The system of claim 18 wherein the LSME has the form:

$$\begin{bmatrix} -T^T T & J \\ J^T & H \end{bmatrix} \begin{bmatrix} m \\ z \end{bmatrix} = \begin{bmatrix} K \\ f \end{bmatrix}$$

where z is a vector containing states, controls and state equation adjoint variables for each time point, grouped by time; m includes adjoint variables for inequality constraints, grouped by time; f and K are vectors; H and J are banded matrices; and T is a diagonal matrix or zero, depending on the quadratic program algorithm selected.